

Calculus Placement Test 2009

University Honors Program

University of Delaware

The purpose of this two-part test is to determine whether you should start in Math 241 (Analytic Geometry and Calculus A) or Math 242 (Analytic Geometry and Calculus B) and to determine whether you qualify for the Honors section of Math 242. **Please complete this test even if you will have AP Math credit.**

- Please mail your completed placement test to :
Susan Katz
University of Delaware Honors Program
207 Elliott Hall
Newark, DE 19716-1256.
- **Your completed test must be postmarked no later than Saturday, June 13, 2009.**
- Please do not use any calculators or computers for this test.
- **Please do not consult anyone when answering the questions. Your solutions should reflect your mathematical skills, so we can determine the course most suitable for your skill level. Students enrolled in a mathematics course for which they lack adequate skills are typically unsuccessful in completing the course.**
- Please show all the steps in your solution. Solutions will be judged on the procedure used to obtain the final answer.
- We advise you to refer to your math notes or a calculus book if you need to. There is no preset time limit and you may take the test over several days. Stop when you feel you have done your best.
- Please write neatly and staple together the sheets numbered 1 to 10. Use extra sheets of paper if you need them.

Name -

UD ID Number -

Calculus proficiency attained (circle all that apply and write your score if you know it)

No calculus AP-AB AP-BC AP-AB Exam AP-BC Exam Other _____

Part I - Test to determine placement in Math 241 or Math 242

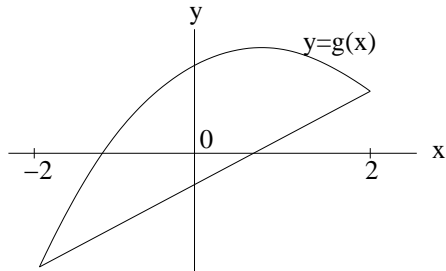
I-1. (a) Compute $\frac{d}{dx} \left(\frac{x}{x^2 - 3} \right)^4$.

(b) Compute $\frac{dy}{dx}$ at $(x=1, y=1)$, where $y(x)$ is the solution of the equation $x^2y^2 + y^7 = 2$ near $(x=1, y=1)$.

I-2. Compute the following integrals -

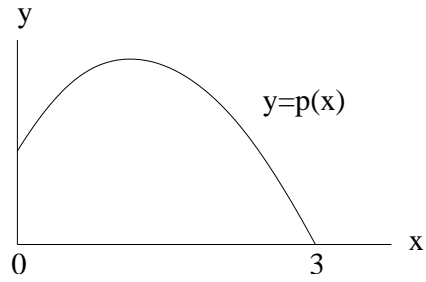
$$(a) \int \frac{t^2}{\sqrt{t-1}} dt, \quad (b) \int_0^{\frac{\pi}{4}} \frac{\tan \theta}{\cos \theta} d\theta.$$

I-3. Compute the area of the region lying below the graph of $g(x) = 3 + 2x - x^2$ and above the line through $(-2, g(-2))$ and $(2, g(2))$.



I-4. The triangular region with vertices $(0, 0)$, $(4, 3)$, and $(4, -1)$ is revolved about the y axis generating a solid. Compute the volume of this solid.

I-5. For each x in the interval $[0, 3]$, consider the triangle with vertices $(0, 0)$, $(x, 0)$ and $(x, p(x))$ where $p(x) = 3 + 2x - x^2$. Find the value of x which generates the triangle with the largest area.



I-6. Sketch a smooth connected curve $y = f(x)$ with the following properties.

$$f(-2) = 8, f(0) = 4, f(2) = 0;$$

$$f'(x) > 0 \text{ if } |x| > 2, \quad f'(x) < 0 \text{ if } |x| < 2, \quad \text{and } f'(2) = f'(-2) = 0;$$

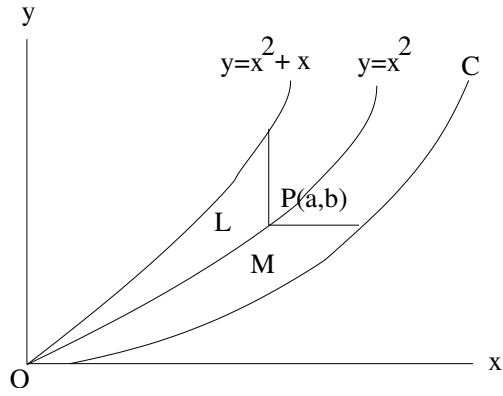
$$f''(x) < 0 \text{ if } x < 0 \quad \text{and} \quad f''(x) > 0 \text{ if } x > 0.$$

Part II - Test to join the Honors section of Math 242

II-1. Find the point on the line $y + 2x = 0$ which is closest to the point (a, b) .

II-2. Find the equation of the curve consisting of all the points P whose distance from the point $(0, 2)$ is twice the distance (of P) from the line $y = 4$. Please simplify your answer as much as possible.

II-3. The figure shows a curve C with the property that, for every point $P(a, b)$ on the middle curve $y = x^2$, the areas of the regions L and M are equal. Find the equation of the curve C .



II-4. A cone of radius 5 cms and height 10 cms is lowered, vertex first, into water in a cylindrical tank of radius 20 cms. The vertex of the cone approaches the bottom of the tank at a speed of 1 cm per second. How fast is the water level rising in the tank when half the height of the cone is submerged?

